

Implications of Supersymmetry Phases for Higgs Boson Signals and Limits

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Abstract

We study the supersymmetry parameter region excluded if no Higgs is found at LEP, and the region allowed if a Higgs boson is found at LEP. We describe the full seven parameter structure of the Higgs sector. When supersymmetry phases are included, $\tan\beta \gtrsim 2$ is always allowed, and the lower limit on M_{H^1} if no signal is found is about 20% lower than in the Standard Model and about 10% lower than in the MSSM with phases set to 0, π .

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I. INTRODUCTION

A general supersymmetric theory has a large parameter space. This is not a major obstacle to study the kinds of signatures to expect because the parameters split up into sets so that only a limited number affect a given process. Once there is data on superpartners the observed patterns will quickly allow us to reduce the number of parameters, just as happened for the standard model in the past.

Early in the history of supersymmetry phenomenology, both for simplicity and in order to learn qualitatively how cross section and decays behaved, simplifying assumptions have been made about the supersymmetry parameters. That was useful. Of course, results based on those assumptions do not hold in general. One purpose of the present paper is to demonstrate that this can have a large effect when non-observation of a signal is used to set limits, or when (hopefully soon) a signal is analyzed to extract information about $\tan\beta$ and the supersymmetry Lagrangian.

The phases of the soft breaking Lagrangian are particularly important. They can play a major role in CP violation, and could provide the CP effects [1] that explain the baryon asymmetry in the universe (which can not be understood in the Standard Model). They could have a major impact on the formulation of how to compactify string theory and break supersymmetry. That the phases can significantly affect the Higgs sector was first pointed out in [2], and independently in [5], and subsequently well studied in [3],..., [10]. Although these studies showed that large effects were possible, two interesting questions were not specifically analyzed. Here we show results for those two questions, which are of considerable interest for the upcoming LEP run and for Fermilab. We understand that a similar analysis of LEP chargino limits will soon appear. [11]

The questions are: (a) If no Higgs boson signal is observed, what is the general lower limit on the lightest Higgs boson mass and the associated limit on $\tan\beta$? (b) If a Higgs boson is observed, what region of the supersymmetry parameters is consistent with the measured mass and $\sigma \times BR$? In both cases we show that the results differ significantly if the phases are fixed at $0, \pi$ as is usual, compared to the situation with general phases allowed.

A complete analysis of the questions can only be done by the experimenters who include efficiencies. We only examine some special cases (which we check are typical) in order to demonstrate the importance of this analysis. Qualitatively, we find that if a Standard Model Higgs boson of mass 105 GeV were excluded, the associated mass limit for the SUSY case with phases 0 or π would be about 95 GeV , and with phases fully included would be about 85 GeV . No LEP Higgs results can exclude values of $\tan\beta$ larger than 2 (the region between 1.5 and 2 is also probably allowed but only for a somewhat narrower parameter space). (Even current results do not exclude $\tan\beta = 2$ when phases are included.) The results of our analysis are shown in Figures 1, 2.

II. FRAMEWORK

A. Higgs Mass Matrix and Mixing

It is known [5] that with one-loop radiative corrections and non-zero CP-violating phase in the MSSM soft Lagrangian, the VEVs of the Higgs doublets can not be chosen to be real simultaneously. Therefore, most generally, we should parameterize the Higgs doublets to be

$$H_1 = \frac{e^{i\alpha_1}}{\sqrt{2}} \begin{pmatrix} v_1 + h_1 + ia_1 \\ h_1^- \end{pmatrix}. \quad (1)$$

$$H_2 = \frac{e^{i\alpha_2}}{\sqrt{2}} \begin{pmatrix} h_2^+ \\ v_2 + h_2 + ia_2 \end{pmatrix}. \quad (2)$$

In the MSSM superpotential and the soft Lagrangian, the only places where the phases α_1 and α_2 appear and can not be rotated away by a redefinition of other matter superfields are the terms $\mu H_1 H_2$ and $m_3^2 H_1 H_2$. Those are also the terms relevant to the scalar potential which determines the vacuum expectation values. therefore in the scalar potential the phases only enter in the combination $\theta = \alpha_1 + \alpha_2$. One can then choose to parameterize the Higgs doublets as

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + h_1 + ia_1 \\ h_1^- \end{pmatrix}. \quad (3)$$

$$H_2 = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} h_2^+ \\ v_2 + h_2 + ia_2 \end{pmatrix}. \quad (4)$$

The neutral part of the Higgs doublets are

$$H_1^0 = \frac{1}{\sqrt{2}} (h_1 + ia_1); \quad H_2^0 = \frac{e^{i\theta}}{\sqrt{2}} (h_2 + ia_2). \quad (5)$$

Define $\tan \beta \equiv v_2/v_1$ (the ratio of the magnitudes of the VEVs). Note that with our choice of parameterization, we can define $\tan \beta$ to be real without loss of generality.

The neutral Higgs potential including one-loop radiative correction is

$$\begin{aligned} V = & \frac{1}{2}m_1^2(h_1^2 + a_1^2) + \frac{1}{2}m_2^2(h_1^2 + a_1^2) - m_3^2[(h_1 h_2 - a_1 a_2) \cos \theta - (h_1 a_2 + h_2 a_1) \sin \theta] \\ & + \frac{\bar{g}^2}{8} D^2 + \frac{1}{64\pi^2} Str \left[\mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right] \end{aligned} \quad (6)$$

where we define

$$D = h_1^2 - h_2^2 + a_2^2 - a_1^2; \quad \bar{g}^2 = \frac{g_1^2 + g_2^2}{4}. \quad (7)$$

Q is the renormalization scale. The supertrace is defined by $Str = \sum_J (-1)^{2J+1} (2J+1)$. \mathcal{M} is the Higgs field dependent mass matrix of particles.

We will only consider the contributions from the top/stop loop and also neglect the contribution from the D-term [7]. From the stop mass Lagrangian

$$\mathcal{L} = [M_L^2 + \frac{1}{2}h_t^2((v_2 + h_2)^2 + a_2^2)]\tilde{t}_L^*\tilde{t}_L + h_t [A_t H_2^0 - \mu^* H_1^{0*}] \tilde{t}_R^*\tilde{t}_L + (L \rightarrow R), \quad (8)$$

we find the mass scalar top mass matrix

$$\mathcal{M}_t^2 = \begin{pmatrix} m_L^2 + \frac{1}{2}h_t^2((v_2 + h_2)^2 + a_2^2) & h_t [A_t H_2^0 - \mu^* H_1^{0*}] \\ h_t^* [A_t^* H_2^{0*} - \mu H_1^0] & m_R^2 + \frac{1}{2}h_t^2((v_2 + h_2)^2 + a_2^2) \end{pmatrix}, \quad (9)$$

where h_t is the Yukawa coupling between Higgs and the top quark which is related to the top quark mass by $h_t = \sqrt{2}m_t/v_2$. From diagonalizing this matrix, we see the phases will only enter the radiative corrections in the combination $\gamma = \phi_\mu + \phi_{A_t} + \theta$, where ϕ_{A_t} and ϕ_μ are the phases of A_t and μ , respectively (as they must by reparameterization invariance).

By minimizing the Higgs potential we get 3 equations. Two of them are obtained by varying the Higgs potential with respect to h_1 and h_2 . There are also 2 equations coming from varying a_1 and a_2 which are actually not independent and yield one nontrivial equation for the phase θ [7,9],

$$m_3^2 \sin \theta = \frac{3h_t^2}{32\pi^2} |\mu| |A_t| \sin \gamma f(m_{t_1}^2, m_{t_2}^2), \quad (10)$$

where

$$f(x_1, x_2) = 2x_1 \left(\log \frac{x_1}{Q^2} - 1 \right) - 2x_2 \left(\log \frac{x_2}{Q^2} - 1 \right) \quad (11)$$

As usual, one can single out the massless Goldstone boson to be eaten by the Z^0 as

$$G^0 = -\cos \beta a_1 + \sin \beta a_2. \quad (12)$$

Now because we have the CP-violating phase in the Higgs potential, the so called pseudoscalar $A^0 = \sin \beta a_1 + \cos \beta a_2$ will mix with the other two neutral scalars h_1^0 and h_2^0 . So in the basis of (h_1^0, A^0, h_2^0) , we will have a 3×3 mass matrix. The mass matrix is real and symmetrical [7,9]. Therefore, it can be diagonalized by an orthogonal transformation

$$\mathbf{U} \mathbf{M}^2 \mathbf{U}^T = \text{diag}(M_{H^1}^2, M_{H^2}^2, M_{H^3}^2), \quad (13)$$

where \mathbf{U} is a real and orthogonal matrix satisfying $\mathbf{U} \mathbf{U}^T = \mathbf{1}$. H^i is the i th Higgs mass eigenstate. The basis of the mass matrix is chosen in such a way that when CP-violation goes to zero, $H^2 \rightarrow A$, the usual pseudoscalar, and H^1 is the lightest neutral scalar.

Let's pause for a moment here to count the relevant parameters. In the original Higgs potential, we have 12 parameters,

$$v_2, v_1, \phi_\mu + \phi_{A_t}, \theta = \alpha_1 + \alpha_2, |A_t|, |\mu|, M_L^2, M_R^2, b = m_3^2, m_{H^1}^2, m_{H^2}^2 \quad \text{and} \quad Q(\text{the renormalization scale}). \quad (14)$$

At the same time, we have the 3 equations obtained by minimizing the potential. So we can use them to eliminate 3 of the parameters. We choose to proceed as following

1. We use two of the extremization conditions (other than the equation for phase θ , Eq. (10)) to eliminate the parameters $m_{H_1}^2, m_{H_2}^2$. We use relation $M_Z^2 = (g_1^2 + g_2^2)(v_1^2 + v_2^2)/4$ and definition $\tan \beta \equiv v_2/v_1$ to rewrite v_1 and v_2 in terms of M_Z^2 and $\tan \beta$. This also inputs one piece of data, the value of M_Z , so one more parameter is eliminated. By doing so, we also make sure the electroweak symmetry breaking occurs appropriately.
2. For any given set of parameters, solve Eq. (10) for θ . So effectively this phase is a function of other SUSY parameters.
3. The renormalization scale Q should not be counted as a parameter; we set it to be 200 GeV . In a more complete analysis, higher order corrections could be used to fix Q so that further loop corrections are minimized.
4. Then we can regard the Higgs mass matrix as depending on the following 7 parameters

$$\tan \beta, \phi_\mu + \phi_{A_t}, |A_t|, |\mu|, M_L^2, M_R^2, b = m_3^2. \quad (15)$$

For any given set of the above 7 parameters, we numerically diagonalize the mass matrix to get the eigenvalues and mixing matrix elements, U_{ij} ; the latter enters into the cross sections and decay branching ratios. (If $\tan \beta$ were large even more parameters could enter from sbottom loops.)

B. The Relevant Lagrangian

We present here briefly the relevant Lagrangian of Higgs production and decay. The Feynman rules can be read from them and the calculation of the production cross section and branching ratios is straightforward.

1. VVH . The Lagrangian is the same for both W and Z bosons.

$$\mathcal{L}_{VVH} = (\cos \beta U_{1i} + \sin \beta U_{3i}) V^\mu V_\mu H^i. \quad (16)$$

2. For the process $Z \rightarrow H^1 H^2$, we have

$$\mathcal{L}_{ZHH} = (\sin \beta U_{2i} U_{1j} - \cos \beta U_{2i} U_{3j}) Z^\mu H^i \overleftrightarrow{\partial}_\mu H^j \quad (17)$$

3. The $H \rightarrow b\bar{b}$ vertex

$$\mathcal{L}_{b\bar{b}H} = \frac{m_b}{v \cos \beta} \bar{b}(U_{1i} - i \sin \beta U_{2i} \gamma_5) b H^i \quad (18)$$

We get a similar expression for the Higgs decaying into a pair of leptons.

4. The $H \rightarrow c\bar{c}$ vertex

$$\mathcal{L}_{c\bar{c}H} = \frac{m_c}{v \sin \beta} \bar{c}(U_{3i} - i \cos \beta U_{2i} \gamma_5) c H^i \quad (19)$$

The branching ratio we are interested in for this paper is $BR = \Gamma(H^i \rightarrow b\bar{b})/\Gamma_{Total}$ where we approximate Γ_{Total} by

$$\Gamma_{Total} = \Gamma(H^i \rightarrow b\bar{b}) + \Gamma(H^i \rightarrow c\bar{c}) + \Gamma(H^i \rightarrow \tau\bar{\tau}). \quad (20)$$

If additional decays can occur, they can be incorporated; the qualitative results will be similar. How one normalizes the observable branching ratios is detector-dependent, so we make a simple choice to illustrate the effects.

III. NUMERICAL RESULTS

We will address the effects of phase on the following two problems:

1. Suppose no Higgs boson is found at LEP. Then in practice there is a limit on $\sigma(e^+e^- \rightarrow ZH^1) \times BR(H^1 \rightarrow b\bar{b})$. Given that limit, what is the general lower limit on M_H^1 in the full 7-parameter space? We compare this with the case of setting the phase to 0 or π as normally done.
2. Suppose a Higgs Boson is found. Its mass and $\sigma \times BR(b\bar{b})$ are measured. What region of the general 7-parameter space is allowed? We compare this with the case when the phase is set to 0 or π as is usually done.

A. Lower Limit of Higgs Mass and $\tan\beta$

If we have an experimental limit on the Higgs production \times branching ratio, we can determine a lower limit on the Higgs mass in the Standard Model. What we are interested in is: if we use the Minimal Supersymmetric Standard Model, including *non-zero* CP violating phases, how will the results one would get in the MSSM without phases change?

To investigate this question, we proceed as follows.

1. Pick a Higgs mass $M_H^{(SM)}$. Then we use the Standard Model to calculate the $\sigma_{prod} \times BR_{(SM)}$. Suppose this calculated $\sigma_{prod} \times BR_{(SM)}$ is in fact the experimental limit, then $M_H^{(SM)}$ is going to be the predicted lower limit on the Higgs mass using the Standard Model.
2. Then we switch to the MSSM. For any given set of parameter we calculate $\sigma_{prod} \times BR_{(MSSM)}$ and the corresponding M_{H^1} . We scan the parameter space. If we find that for certain parameters, $\sigma_{prod} \times BR_{(MSSM)} < \sigma_{prod} \times BR_{(SM)}$ (The experimental limit) and at the same time $M_{H^1} < M_H^{(SM)}$ (the lower limit predicted by Standard Model), then we get a potential candidate for a new lower limit of Higgs mass.
3. In order to achieve the above requirement, one must find a set of parameters such that the coupling of H^1 to Z is smaller than in the Standard Model. However, this is potentially dangerous. When the coupling for ZZH^1 decreases, the coupling for process $Z \rightarrow H^1 H^2$ increases. So we also need to make sure that the cross section

for $Z \rightarrow H^1 H^2$ times its branching ratio to $b\bar{b}b\bar{b}$ won't get so large that it would have been seen. This is a complicated issue related to the b -tagging efficiency. Since our purpose is to illustrate typical effects rather than to settle the issue, we will make a crude estimate and require that $\sigma_{Z \rightarrow ZH^1} \times BR(H_1) > \sigma_{Z \rightarrow H^2 H^1} \times BR(H_1) \times BR(H_2)/3$. The factor of $1/3$ is assumed to reflect efficiencies.

The seven dimensional parameter space makes it difficult to illustrate the most general results. Consequently we simply illustrate with a few parameters fixed to show the effect of the CP-violating phase. We emphasize that we do not in any sense argue that the parameters we fix should be thought to have those actual values. We have checked that the results we present are typical. The results are shown in Figure 1. We see that if we allow a non-zero phase, we get a new potential lower limit on the Higgs mass around 85 GeV . And lower $\tan\beta$ values become allowed. If the phase is 0 or π , the usual results emerge.

B. MSSM Parameter Region

First we pick a set of parameters to generate a set of 'fake' data (a cross section \times branching ratio and a mass). Then we take into account estimated experimental errors (10% for cross section and 5% for mass) and find the range of parameters that can generate such a set of data. Again, to illustrate the effects of phases, it suffices to focus on a typical region of the parameter space. So we will fix $M_{\tilde{L}}$, $M_{\tilde{R}}$ and μ at the values used to generate the fake data and allow A_t , $\tan\beta$ and m_3 to vary. We compare the results from fixing the phase to be 0 , π with the results allowing the phase to vary. We illustrate the results by showing the allowed region in the $\tan\beta$ and $2m_3^2/\sin 2\beta$ plane in Figure 2; ($2m_3^2/\sin 2\beta$ would be M_A in the absence of CP violation).

We see the presence of phases allows an enlargement of the parameter region and in particular it allows lower values of $\tan\beta$.

IV. CONCLUSION

If nature is indeed supersymmetric, the soft supersymmetry breaking Lagrangian is the basic thing to be determined. Masses of mass eigenstates and cross sections \times branching ratios are measured, and have to be converted into the parameters of the Lagrangian. The patterns in the resulting Lagrangian will help (and may be essential) to formulate how to convert 11D supersymmetric M-theory into a 4D theory with broken supersymmetry, so that M-theory becomes testable in the normal sense.

Extracting the parameters of the Lagrangian must be done with full generality. In this analysis we have shown that including the supersymmetry phases (that are usually ignored) can have qualitative large effects on how both the presence and absence of a signal for a Higgs boson are interpreted. Similar effects will occur for superpartner limits.

The results are in the two figures. Qualitatively, we find that the excluded Higgs boson mass is about 20% lower than it would be in the Standard Model and about 10% lower than it would be in the MSSM without phases, and that $\tan\beta$ values down to 2 or even lower are

always allowed by LEP results. We have focused on LEP, but a similar analysis is required for Fermilab and LHC.

V. ACKNOWLEDGMENTS

We appreciate helpful comments from M.Brulik, L.Everett, S.Mrenna, and J.Wells.

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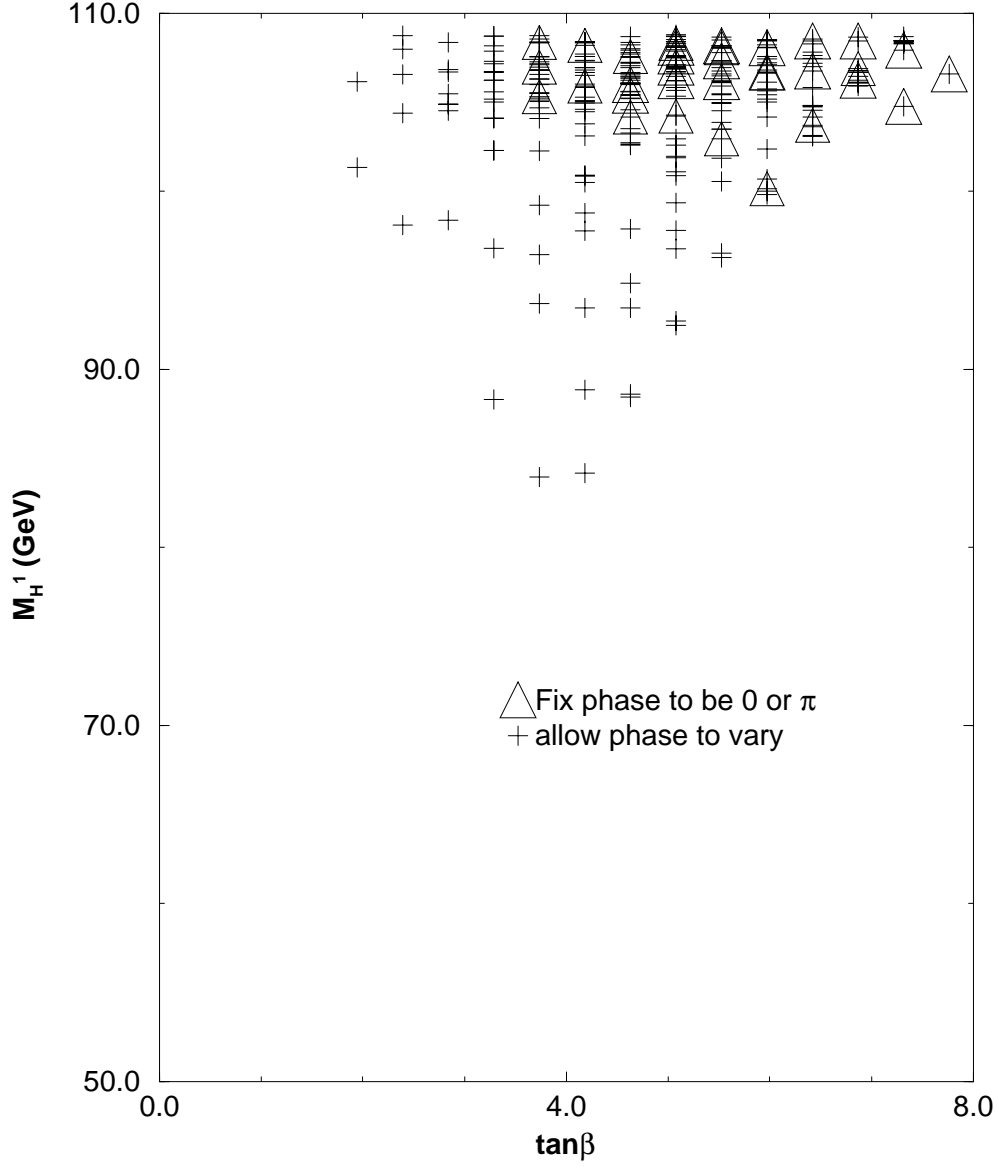


FIG. 1. The allowed MSSM parameter region if no Higgs has been found. We set $M_{\tilde{L}} = 300\text{GeV}$, $M_{\tilde{R}} = 250\text{GeV}$. We pick two points in the (A_t, μ) space: $A_t = 500\text{GeV}$, $\mu = 300\text{GeV}$ and $A_t = 350\text{GeV}$, $\mu = 250\text{GeV}$. We vary $50\text{GeV} < m_3 < 100\text{GeV}$, $0 < \phi_\mu + \phi_{A_t} < \pi$ and $2 < \tan\beta < 10$. We use $M_H^{(SM)} = 105\text{GeV}$. The \triangle s show the region of this plane would be allowed if the phase were fixed at 0 or π , while the $+$ show that a significantly different region is allowed for the full acceptable range of the phases

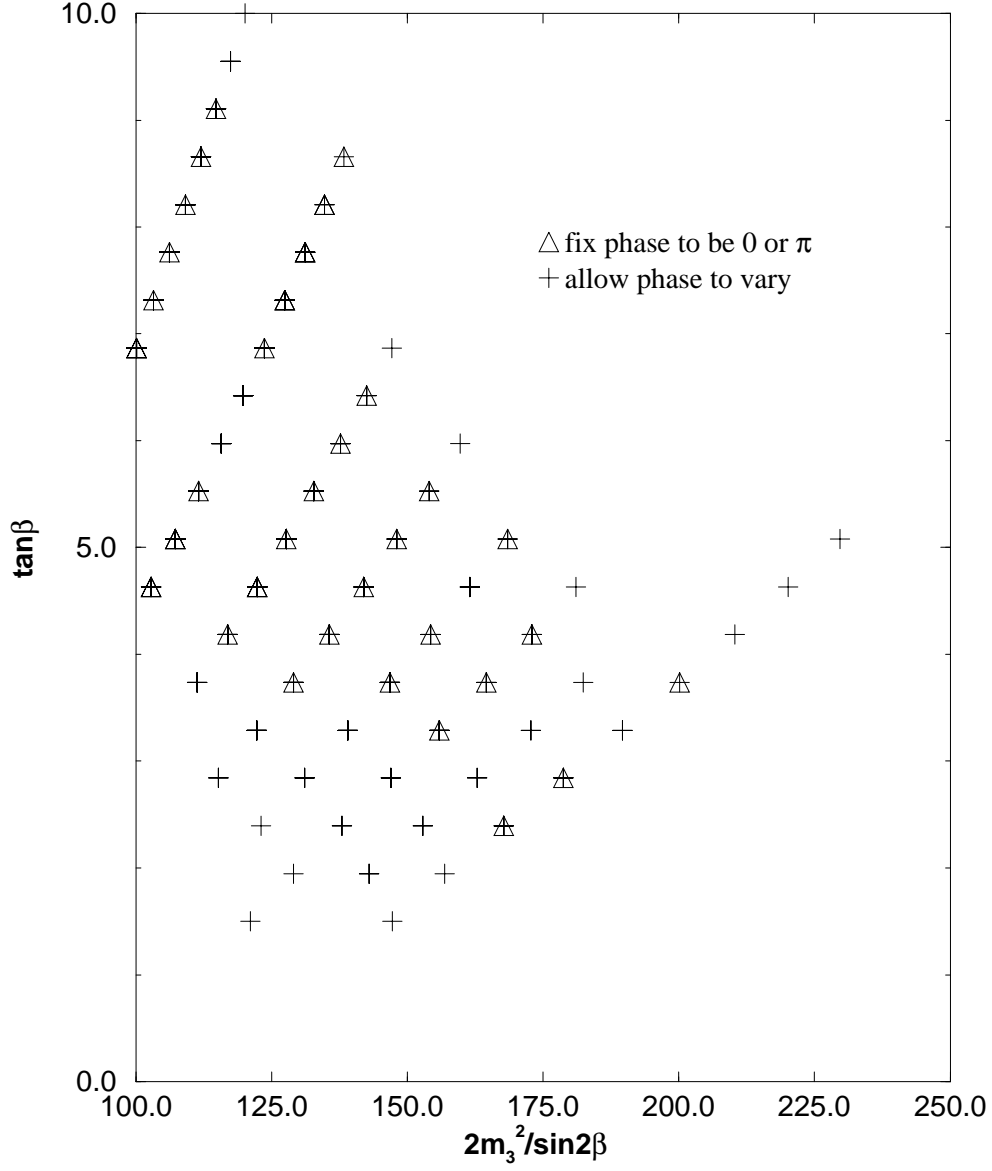


FIG. 2. The allowed MSSM parameter region if one neutral Higgs has been found. Fake data generated by $M_{\tilde{L}} = 300\text{GeV}$, $M_{\tilde{R}} = 250\text{GeV}$, $A_t = 500\text{GeV}$, $\mu = 300\text{GeV}$, $\tan\beta = 6.0$, $\phi_\mu + \phi_{A_t} = 0.0$ and $m_3 = 50\text{GeV}$. The horizontal axis would be M_A if there were no CP violation in the Higgs sector. The results show that a larger region of $\tan\beta$ and " M_A " is allowed if the effects of phases are included